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Vagueness in Philosophy – "Unbestimmtheit" in Physics

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I. Introduction

Since Eubulides the problem of vagueness has existed in philosophy. Vague predicates do not allow one to decide whether the predicate is true or false. The Sorites paradox demonstrates such a borderline case. Do ten rice grains form a heap or do they not form a heap? What happens if we have a grain less? There has been an extensive discussion of vagueness[1] in modern philosophy[2] which I will not be able to discuss in detail. However, in order to work out the difference between vagueness in philosophy and Unbestimmtheit in physics, a short summary of what our colleagues in philosophy have been concerned with is necessary. Modern philosophers have been very interested in language. Both vagueness and precision are inherent to representations.[3] A large-scale map is less accurate or more vague on small roads than a small-scale map which also shows these details. Words as parts of a representation system are often vague in everyday language. Take the word "red". It includes all kinds of shades from a yellowish red to a deep purple. Philosophers are concerned with vagueness in propositions since a vague proposition does not allow a decision as to whether it is true or false. There have been various attempts to solve this problem. Many-valued or fuzzy logic with more truth values has been tried. Supervaluationism gives a special value to a subset of statements where any sharpening of the language does not pose problems. This procedure cannot deal with borderline cases which arise in any practical procedure. Consider the following example.

At the start of the semester new students are accepted. There are some who do fulfil all the requirements of acceptance and others who do not fulfil these demands. Besides these cases there are also a number of borderline cases which do not belong to either of the two categories.

So, what should one do with these borderline cases? Are they a consequence of our reduced ability to judge? A bibliography of the literature on vagueness and the Sorites Paradox can be found on the web.[4]

Complementary to this discussion in philosophy, I compiled cases of "Unbestimmtheit" in physics. Unbestimmtheit includes uncertainty and indeterminacy.

The concept “uncertainty” has various meanings in physical literature. It refers to the lack of knowledge of an observer, the experimental inaccuracy with which a quantity is measured or to the spread of an observable in an ensemble of similar systems. Indeterminacy or indefiniteness indicates the absence of a boundary, as was discussed in the example of the acceptance procedure of new students where some borderline cases always arise. Quantum indeterminacy expresses the “ontic” or factual Unbestimmtheit of quantum events, whereas quantum uncertainty only states our lack of knowledge. These are two different interpretations of quantum mechanics. I will later argue that in my opinion, quantum indeterminacy is the correct interpretation which expresses that the underlying quantum fluctuations are a property of nature and not due to a set of hidden variables which we do not yet know. My original motivation was to look around in the everyday physics world. It was in this way that the separation into factual and theoretical Unbestimmtheit emerged. This provides rather a phenomenology than a conceptual differentiation. I wanted to show how well we physicists handle uncertainty. We can even do calculations in “uncertain” circumstances. The advantage is that physicists have developed a scientific language with stronger rules than everyday language to attack this problem. The intention of this short article is to show that in the special case of one scientific discipline, the problem is more concrete than the general debate in philosophy and therefore has also evolved towards more concrete solutions.

II. Factual Unbestimmtheit

I understand as facts observations or phenomena which are part of nature itself or of the way we investigate nature. They are independent of the models with which we describe nature. This view may be called naive realism at this stage.

A. Experimental Errors

As an empirical science, physics must deal with inaccuracy in experiment and observation. Ideally, an experiment delivers one or more numbers which are called measurement values. We have to differentiate between experiments which examine so-called constants of nature, which are not dependent on time and the specific circumstances of their determination, and e.g. properties of materials which can depend on their preparation and purity. In the first case we assume that these constants of nature have a definite value. In the second case the material probe has to be reproduced exactly in order to create the same object with the same measurable properties. Non-ideal detectors or apparatus introduce additional measuring errors. In general, independent experimental measurements will give different results. However, the mean of a measurement and the fluctuations around this mean, the variance σ, can be determined. For the testing of theories, deviations of a parameter by (3-4) σ from its theoretically predicted value are relevant; which means that they are so improbable that the theory can be rejected. Boundary cases arise when the measuring value neither confirms nor rejects the theoretical hypothesis; this typically means there are deviations of (2-3) σ. In general, the behaviour of a physicist is then conservative. He assumes that the experimental result does not contradict the theory.

B. Natural Boundary Cases

A measurement gives neither a big nor a small number; it needs a scale on which one can compare different measurements. Here it is worthwhile to refer to the long and tedious discussion of the baldness problem and/or the Sorites paradox in the philosophical literature. Obviously, the question of calling a man bald emerges after having examined his head. Assuming that we have found a thick head of hair, we are not motivated to and probably cannot count the number of individual hairs. Consequently, our statement about the baldness of the person will be vague. In physics, counting or weighing or reading a digital meter will always be at the root of avoiding vagueness. But according to the analytical philosophers this does not help. Even if we measure the richness of a person exactly in € and cents we still cannot fix the amount X where people start to be rich. A person with (X minus 1000) € yearly income is still rich. Repeating this step several times leads to nonsense which demonstrates the Sorites paradox. The boundary cases between the rich and the poor are numerous. One way out is an analysis of the income distribution which shows that it deviates from a Log-normal distribution at high incomes and follows a power law for “really” rich people. Therefore one can associate richness with the crossing of these two qualitatively different distributions.

Let us discuss boundary cases in more detail. If the correct representation of a physical quantity is discontinuous, then it is possible that there are boundary cases which belong neither to one nor to the other group. To this purpose, consider the periodic system. The atomic weight of plutonium is given as 195.09 in units of the carbon atom 12. This approximate and unusual number arises if one weighs plutonium. Neutral plutonium 195 has 78 protons, the same number of negatively charged electrons and 117 neutrons. How does the non-integer number 195.09 come about? Chemistry is orienting itself according to the nuclear charge or the number of electrons. The natural way to handle this problem is to look for another coordinate which organizes atomic nuclei and which goes beyond the chemical properties of atoms.

Nature allows an atomic nucleus with the same nuclear charge, the same number of protons, i.e. with the same chemistry to have a different number of uncharged neutrons. In most cases the number of neutrons is 117. But there exist isotopes with more neutrons. This explains the non-integer atomic number 195.09. The hypothesis that there exists an unknown coordinate or an as yet not understood parameter is characteristic for scientific thinking if it tries to handle non-categorizable boundary cases.

C. Quantum Physical Indeterminacy

There has been a long-standing discussion about the interpretation of quantum physics.[5] The basis of this discussion is the Heisenberg uncertainty relation which states that the position Δx and momentum Δp of a particle cannot be measured with arbitrary exactitude, $\Delta x\, \Delta p > h/(4\pi)$, where h is the Planck action quantum. This is a statement of indeterminacy which is founded in nature itself and does not depend on the quality of our measuring apparatus. The particle is described by a wave function which considers the particle as a superposition of localized states. This wave function belongs to the particle and gives a sort of “sleeping”-state of the particle which will be realized with probability $|\Psi(r)|^2$ when one makes a measurement at the position r. This wording “sleeping-state” should be taken metaphorically, it is meant to express the fact that the wave function cannot be observed. There is a debate about whether the wave function can be identified with the information content of the particle.[6] The broader the wave function in position/coordinate space the smaller the transformed wave function in momentum space. This mathematical transformation is related to the uncertainty relation. In quantum physics we have well-defined rules to handle these uncertainties. Quantum physics gives a tight framework for consistent calculations of these uncertainties. The measuring process has been explained[7] as the result of the interplay of the system, the measuring apparatus and the environment. There is no doubt about the importance of decoherence in this triad, i.e. the loss of phase relations in the superposition of combined system-apparatus states. This decoherence has also been experimentally demonstrated. More subtle is the proposed mechanism for how, in the multitude of possible product states, only certain “pointer states” survive, which allow a reading of the pointer in the measurement apparatus. The above authors argue that it is the apparatus-environment interaction which makes only those states survive via the Heisenberg indeterminacy condition which commute with the operators monitored by the environment. All other states will not leave a record.

One of the challenges of modern physics is to discuss if or how one should include gravity in the framework of quantum mechanics. A fundamental role in quantum gravity is played by the Planck scale. If such an elementary Planck scale exists, it would be impossible to measure anything better than this Planck length. Every measurement of a position along the x-direction would be associated with an uncertainty in the other direction, i.e. the y-position. One has the case of a non-commutative geometry which declares position coordinates to be non-commuting variables. A binary operation is called non-commutative when the result of the binary operation depends on the order of the inputs. For example, the addition of two numbers is commutative $a+b=b+a$, but the subtraction is not $a-b \neq b-a$. In quantum mechanics we call two operators A and B non-commuting when their multiplication $A\,B \neq B\,A$. One may imagine that A and B have the character of matrices. Since the theory of quantum gravity has not yet been formulated, it is not the task here to dwell on it. An analogy in quantum mechanics which is, however, well known is the behaviour of the x- and y-coordinate of an electron in a homogeneous magnetic field along the z-direction. The quantum mechanical x- and y- coordinates do not commute in this case. The role of the Planck length squared is given by the action quantum squared divided by the mass of the electron and cyclotron frequency.

Consequently, the centre of the electronic orbit cannot be measured with arbitrary high exactness. If one goes in quantum mechanics to high and higher energy, one can test smaller and smaller distances. This is not the case in a theory with a fundamental shortest length. Interesting effects will be hidden to us and remain behind the horizon of the Schwarzschild radius $R=2\ G\ E/c^2$. In a collision at high energies a black hole will be produced which can only emit low-energy Hawking radiation the energy of which is of the order of the inverse Schwarzschild radius. A new uncertainty relation arises in this hypothetical world of quantum gravitation. This speculation is an extension of our experience with quantum theory, which may turn out to be true. Why should one bother with it? I think this case shows a decisive effect produced by “Unbestimmtheit” in physics. Unbestimmtheit is not an obstacle to research, but a structuring feature of scientific progress. The as yet undetermined part of things around us triggers the theoretical fantasy of the physicist who finds guidance in mathematical structures like non-commutative geometry.

III. Theoretical Unbestimmtheit

Besides the factual Unbestimmtheit there is theoretical indeterminacy as a key ingredient of physical models. The differentiation I make here is the differentiation of the everyday physics world. Factual Unbestimmtheit is associated with the laboratory where the physicists in white coats work on equipment and do real measurements. Theoretical Unbestimmtheit is to be found in the offices of the theoretical physicists. This separation of the discipline occurred in most subfields at the end of the 19th century. In some new and evolving subfields it is not yet very strong – astrophysicists sometimes claim both fields. Theoretical physicists deal with models. In principle, quantum gravity would be such a model when it is mature and well defined. So-called statistical models of physics start with undetermined micro states which are assumed to form a statistical ensemble defined by the knowledge of a few macroscopic variables. The statistical method in physical theories has made out of a defect a virtue. When one considers the dynamics of a gas even in classical mechanics, one cannot determine all particles’ positions and momenta. There are just too many of them.

A. Uncertainty in Statistical Physics

Boltzmann developed the hypothesis of atoms by his theory of statistical phenomena. He was able to enumerate all micro states which belong to a given macro state. The macro state is described by physical properties like temperature, volume and particle number which can be easily measured. Boltzmann was able to give statistical meaning to the entropy which encodes the lack of information about the system. Lack of information means uncertainty about the micro states. We do not know the individual positions and velocities of all the atoms. The less we know about the micro states the higher the entropy. Entropy can be compared with negative actual information. By being able to calculate with high accuracy systems of many particles and comparing calculations with slightly different initial conditions, physicists found that they yield totally different final results. The results of these calculations are chaotic. The physics of chaos has attracted a lot of attention in the last decades, since it is intimately connected with our possibilities to predict future events. A small uncertainty in the initial conditions leads to an extremely big uncertainty in the final results. To take into account this uncertainty means to map out these dependencies and not rely only on the deterministic classical dynamics.

B. Uncertainty in Biophysical or Econophysics Problems

In 1827 the English botanist Brown observed pollen in a liquid under the microscope. He saw that the pollen moved in a totally random fashion, as if it were a living being. It took almost another 70 years until Einstein could explain this phenomenon. His theory of Brownian motion opened up the possibility to understand stochastic processes. The light particles in the liquid transfer momentum to the heavy particles and push them around in a stochastic manner. The forces which they exact cannot be determined. These forces are even zero in the mean. Nevertheless, the pollen moves. The reason behind this movement is that in a certain interval of time, the forces are correlated with each other. They will not change abruptly from large negative to large positive values but the magnitude of forces will show some correlation in time. The physicist fills the grey zone of uncertainty by postulating an autocorrelation function in time. Einstein’s solution leads to a mean quadratic velocity which for large times becomes proportional to the strength of the correlation functions.

Differential equations with externally stochastic terms are used to model complex biological or economical problems. Physics is able to model phenomena which will never be known in full certainty. A combination of probability theory and differential calculus helps to understand random systems in a better way. There is a special case of Brownian motion where the fluctuations are driven by quantum behaviour,[8] e.g. “the tunnelling and the transfer of electrons or other quasi particles in solids is assisted by noise for which the quantum nature cannot be neglected. The features of this noise change drastically as a function of temperature. At sufficiently high temperatures a crossover does occur to classical Johnson-Nyquist noise.” This example demonstrates that the delicate differentiation between uncertainty (our lack of knowledge) and indeterminacy (intrinsic quantum property) can be mixed up in reality.

C. Indeterminacy in Quantum Stochastic Models

The statistical treatment of middle-sized quantum systems gives rise to new problems. The number of particles in these systems is small compared to thermodynamic systems. We have 100-200 particles only, in comparison with 10^{23} particles. In addition, the system itself has a small size, therefore we have to use the laws of quantum physics and must handle statistics in some other way. Quantum objects of this kind are atomic nuclei with excitation energies of a couple of mega-electron volt or quantum billiards in two dimensions in solid state physics. A theoretical treatment of these systems can elucidate the uncertainty, i.e. describe certain aspects of the energy spectrum. Modern methods are based on a theory which handles, instead of a single quantum mechanical energy matrix, a class of energy matrices which are only limited by symmetry properties. The lack of knowledge in this case is fully connected to theoretical modelling. Physics cannot parameterize the complex interactions of the few particles in detail. The model itself is fully quantum mechanical, i.e. in this respect indeterminate. The successful method is to model a statistical distribution of random matrices which contain the main symmetry properties of the problem. Please note how the theoretical physicist structures a problem which may be considered “unbestimmt”. The symmetries are necessary in order to limit the number of possible boundary cases and then subclasses of uncertain cases can be connected with subclasses of phenomena. It was even proposed to apply stochastic theory to understand the fundamental form and coupling constants of the standard model of elementary particles. In this approach the standard model arises as the result of stochastic averaging over complicated interactions.[9]

The proponent of this “Random Dynamics” H. B. Nielsen explains, “In the search for the most fundamental theory of physics one usually looks for a simplest possible model, but could it not be that the fundamental “World Machinery” (or theory) could be extremely complicated? We see that we have some very beautiful and simple laws of nature such as Newton’s laws, Hooke’s law, the Standard Model and so on – how could such transparency and simplicity arise from a very complex world? The Random Dynamics project is based on the idea that all known laws of nature can, in a similar way as Hooke’s law, be derived in some limit(s), practically independent of the underlying theory of the World Machinery. The limit which could suggestively be the relevant limit for many laws, would be that the fundamental energy scale is very big compared to the energies of the elementary particles even in very high energy experiments. A likely fundamental energy scale would be the Planck energy, 1.2 10^{19} GeV.[10] This is an extreme approach to elementary particle research which is singular among high-energy physicists.

IV. How should one handle Unbestimmtheit?

A. The Task of Clarification

Pragmatically, the importance of an uncertain result has to be assessed in the context of the physical model or theory. There are uncertain results which do not have to be improved because nobody really can give a reason for more exact measurements. Uncertainties may be flatly uninteresting as the research of extra sensorial processes has shown. Not everything which deviates from the expected probabilities in everyday life must be scientifically researched. However, results which trigger an important direction of the theoretical development are cases in which every scientist will be keen to improve the result as quickly as possible. In this case the experimental physicist has a high responsibility to start an investigation. This may be common wisdom within the physics community, but in the general public the singular theory driven physicist has become the icon of the physicist. Einstein invented General Relativity Theory without any experimental hints. In a similar way the string community claims that the time is ripe for another venture based purely on theoretical beauty and rigor.

B. Classify Boundary Cases in New Generic Categories

In physics the concept of atomic weight is nothing uncertain, but can lead to boundary cases which are not comprehensible if one does not know the content of the atomic nucleus. The result of a measurement leads to uncategorizable boundary cases. One must find the new category, in this case the category neutron number to clarify these uncertain boundary cases. Similarly for Hamiltonian random matrices or in the handling of stochastic differential equations one must recognize the intention to give structure to our lack of knowledge by constructing symmetry classes of random matrices which then lead to similar phenomena. The modelling of biological systems has also led to the invention of generic classes which give exemplary structures even without knowledge of individual parameters. Here the modelling of Brownian motion was demonstrated as an example. The theory of disordered systems has clarified this area in an important way which is still under investigation in neurophysiology.

C. Define Limits

Modern physics tries to spell out structural indeterminacy in detail. This has been successful in quantum mechanics. Ontic indeterminacy which is based in nature itself leads to theoretical constructs which have been highly productive. I see new developments in this direction in the theoretical work concerning the uncertainty of space-time, where non-commutative geometry can play a theoretical role which leads to other theoretical consequences about black holes and cosmology.

D. "Fuzzy" Logic

The engineering scientist encounters the problem that the time for a decision is limited, therefore machines have to make a decision in a situation which

is only vaguely defined, e.g. by a vague predicate. Here the mathematical branch of "fuzzy" logic has been established which constructs weighted statements leading to decisions in any case. The focus of this method is to assign each vague concept a membership function and then transform the vague rules into a mathematical algorithm to calculate the decision. This procedure is highly successful in control situations, e.g. controlling the pressure or the temperature or the velocity of a technical object. The discussion in philosophy has focused on whether the introduction of half-true values for these membership functions is justified. In my opinion, it should, however, concentrate more on the question of how to use expert knowledge to encode membership functions and how to establish the rules with which they are manipulated. These are objects of criticism with full justification.

There are some similarities between the discussion about "fuzzy" logic and the discussion about quantum logic, in which the principle of "tertium non datur" is supposedly violated due to the indeterminacy of quantum mechanics. Take the double slit experiment, where the detector pattern shows interference. When we define the measurement of the electron as A, the passage of the electron through the upper slit as B and the passage of the electron through the lower slit with $\neg B$ (non B), then the probability statement of quantum mechanics has the form:

$$p(A)=p(A\cap B)+ p(A\cap\neg B)+ \text{interference term},$$

whereas we would naively expect that completeness gives $p(A)= p(A\cap B)+ p(A\cap\neg B)$. The quantum behaviour violates that there is an alternative to B and $\neg B$ for probabilities, but I see no reason to introduce a "quantum logic" which deviates from the two-valued classical logic.

In the practical sciences, one must differentiate between urgent cases of decision-making and those where time is not an important factor. In these other cases, decisions founded only on a mathematical algorithm become themselves open to criticism. Uncertain cases should lead to renewed thought, fresh research and empirical investigation before a new agenda is decided upon. Since these boundary cases typically emerge between major disciplines, connected and neighbouring concepts have to be considered, i.e., the full network surrounding them has to be identified. This wider scope generates new insights in cases of otherwise undecidable borderline situations.

V. Summary

Based on a list of uncertain and indeterminate cases in physics I have complemented the discussion about vagueness in philosophy. The scientific framework of physics tries to avoid vague concepts which appear often in everyday language. Measurement is the principle tool for specifying physical results. But, nevertheless, lack of knowledge enters the experimental and theoretical process of physical science. In experiment we encounter measurement errors which can lead to boundary cases where confirmation or refutation of a theoretical model becomes confused. Boundary cases can sometimes be avoided by a new concept which explains why no clear-cut separation has been seen before. The quantum domain presents ontic indeterminacy where the physicist has no possibility to better the situation. In theory, lack of knowledge has been turned into a virtue with the help of statistical concepts which classify systems by their macroscopic similarities assigning probabilities to their microscopic makeup. Forecasting thereby also becomes possible with limited accuracy. Special tools like fuzzy logic are more appropriate to rule-based practical sciences than to the natural sciences.

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